

(imp)

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$(a + b - c)^2 = a^2 + b^2 + c^2 + 2(ab - bc - ca)$$

$$(a - b + c)^2 = a^2 + b^2 + c^2 + 2(-ab - bc + ca)$$

$$(-a + b + c)^2 = a^2 + b^2 + c^2 + 2(-ab + bc - ca)$$

$$x^2 = (-x)^2$$

$$(-a+b-c)^2 = (a-b+c)^2$$

$$(a + b + c)^2 = \underbrace{(a^2 + b^2 + c^2)}_{?} + 2 \underbrace{(ab + bc + ca)}_{??}$$

$$a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca)$$

$$(ab + bc + ca) = \frac{(a + b + c)^2 - (a^2 + b^2 + c^2)}{2}$$

29. The algebraic expression $25x^2 + 16y^2 + 9z^2 - 40xy + 24yz - 30zx$ is equal to:

बीजीय व्यंजक $25x^2 + 16y^2 + 9z^2 - 40xy + 24yz - 30zx$ निम्नलिखित में से किसके बराबर है:

☒ [A] $(4y + 3z - 5x)^2$

[B] $(4y - 3z - 5x)(4x + 5y + 3z)$

[C] $(4y + 3z - 5x)(4x + 5y - 3z)$

[D] $(4y - 3z - 5x)^2$

$$= (-5x + 4y + 3z)^2$$

30. What is the value of $a^2 + 4b^2 + 9c^2 - 4ab + 12bc - 6ca$?

$a^2 + 4b^2 + 9c^2 - 4ab + 12bc - 6ca$ का मान क्या है?

SSC CHSL TIER – I 2022

[A] $(a + 2b - 3c)^2$

[B] $(2a - b - 3c)^2$

[C] $(2b - a - 3c)^2$

✓ [D] $(a - 2b - 3c)^2$

$$\begin{aligned} &= (-a + 2b + 3c)^2 \\ &= (a - 2b - 3c)^2 \end{aligned}$$

$$(+5)^2 = (-5)^2 = 25$$

31. If $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8zx = (Ax + y + Bz)^2$, then the value of $(A^2 + B^2 - AB)$ is:

यदि $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8zx = (Ax + y + Bz)^2$ है, तो $(A^2 + B^2 - AB)$ का मान है:

[A] 16

[B] 14

[C] 6

[D] 18

$$2 + 8 - (-4) = 14$$

$$(-\sqrt{2}x + y + 2\sqrt{2}z)^2 = (Ax + y + Bz)^2$$

$$-\sqrt{2} = A$$

$$2\sqrt{2} = B$$

32. If $\frac{a^2+b^2+c^2-1024}{ab-bc-ca} = -2$ & $a+b=5c$, where $c>0$, then the value of c is?

यदि $\frac{a^2+b^2+c^2-1024}{ab-bc-ca} = -2$ & $a+b=5c$ है, जहाँ $c>0$, तो c का मान है?

[A] 8

[B] 4

[C] 12

[D] 5

$$a+b-c=32$$

$$4c=32$$

$$c=8$$

$$a^2+b^2+c^2-1024 = -2ab+2bc+2ca$$

$$a^2+b^2+c^2+2ab-2bc-2ca = 1024$$

$$(a+b-c)^2 = 1024$$

33. If a, b, c are non-zero real numbers and $a^2 + b^2 + c^2 + 2ab - 2bc - 2ca = 0$, then find the value of $\frac{6(a+b+c)+2c}{(a+b+c)}$?

यदि a, b, c गैर-शून्य वास्तविक संख्याएं हैं और $a^2 + b^2 + c^2 + 2ab - 2bc - 2ca = 0$, तो $\frac{6(a+b+c)+2c}{(a+b+c)}$ का मान ज्ञात कीजिये?

[A] 1

[C] 6

[B] 8

[D] 7

$$\frac{4c}{2c} = 2$$

$$(a+b-c)^2 = 0$$

$$a+b=c$$

34. If $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = \sqrt{3}$ and $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 0$, then find the value of $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$ (Where x, y, z, a, b, c are non zero numbers)

यदि $\left(\frac{x}{a}\right) + \left(\frac{y}{b}\right) + \left(\frac{z}{c}\right) = \sqrt{3}$ और $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 0$ तो $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$ का मान ज्ञात कीजिए। (जहाँ x, y, z, a, b, c शून्येतर संख्याएँ हैं)

[A] 0

[B] $\sqrt{3}$

[C] 3

[D] 6

$$a^2 + b^2 + c^2$$

35. If $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = 11$ and $\frac{b}{a} + \frac{c}{b} + \frac{a}{c} = 8$ then find the value of $\frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2}$?

यदि $\left(\frac{a}{b}\right) + \left(\frac{b}{c}\right) + \left(\frac{c}{a}\right) = 11$ और $\frac{b}{a} + \frac{c}{b} + \frac{a}{c} = 8$ है, तो $\frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2}$ का मान ज्ञात कीजिए।

RRB NTPC 2021

[A] 105

[B] 19

[C] 137

[D] 75

$$a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca)$$

$$\begin{aligned} \left(\frac{a}{b}\right)^2 + \left(\frac{b}{c}\right)^2 + \left(\frac{c}{a}\right)^2 &= \left(\frac{a}{b} + \frac{b}{c} + \frac{c}{a}\right)^2 - 2\left(\frac{a}{c} + \frac{b}{a} + \frac{c}{b}\right) \checkmark \\ &= 11^2 - 2 \times 8 \\ &= 105 \end{aligned}$$



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36. If $x + y + z = 22$ and $xy + yz + zx = 35$, then what is the value of $(x - y)^2 + (y - z)^2 + (z - x)^2$?
 यदि $x + y + z = 22$ तथा $xy + yz + zx = 35$ है, तो $(x - y)^2 + (y - z)^2 + (z - x)^2$ का मान क्या है?

[A] 793

[B] 681

[C] 758

[D] 715

$$= 2x^2 + 2y^2 - 2xy$$

$$= 2 \cdot (x^2 + y^2 - xy)$$

$$= 2 \times (44 - 35)$$

$$=$$

1-पर
 extra $\rightarrow$ x, y, z
 but $z=0$

$$x + y = 22$$

$$xy = 35$$

$$\begin{aligned} x^2 + y^2 &= (x + y)^2 - 2xy \\ &= 484 - 70 \\ &= 414 \end{aligned}$$

37. If $(a + b + c) = 17$, and $(a^2 + b^2 + c^2) = 115$, find the value of $(a + b)^2 + (b + c)^2 + (c + a)^2$.
यदि $(a + b + c) = 17$, और $(a^2 + b^2 + c^2) = 115$, तो $(a + b)^2 + (b + c)^2 + (c + a)^2$ का मान ज्ञात कीजिए।

[A] 402

[B] 404

[C] 408

[D] 394

$$= 2 \cdot (a^2 + b^2 + ab)$$

$$= 2 \times (115 + 87)$$

$$= 404$$

put $c = 0$

$$ab = \frac{17^2 - 115}{2} \\ = 87$$

38. If, $(a - b)^2 + (b - c)^2 + (c - a)^2 = 6$ and $a^2 + b^2 + c^2 = 29$, then what is $(a + b + c)$ equal to?

यदि $(a - b)^2 + (b - c)^2 + (c - a)^2 = 6$ और $a^2 + b^2 + c^2 = 29$ है, तो $(a + b + c)$ किसके बराबर है?

(UPSC CDS - II 2025)

[A] ± 9

[B] ± 8

[C] ± 6

[D] ± 3

put $c=0$

$$2(a^2 + b^2 - ab) = 6$$

$$\underbrace{a^2 + b^2}_{29} - ab = 3$$

$$a^2 + b^2 = 29$$

$$ab = 26$$

$$= \pm \sqrt{a^2 + b^2 + 2ab}$$

$$= \pm \sqrt{29 + 52}$$

$$= \pm 9$$

39. If $x + y + z = 19$, $x^2 + y^2 + z^2 = 133$, and $xz = y^2$, then the difference between z and x is:

यदि $x + y + z = 19$, $x^2 + y^2 + z^2 = 133$, और $xz = y^2$, तो z और x के बीच का अंतर है:

[A] 5

[B] 3

[C] 6

[D] 4

$$4 \times 9 = 6^2$$

imp data $4, 6, 9$

$$4 \times 9 = 6^2$$

$$\left. \begin{aligned} 4 + 6 + 9 &= 19 \\ 4^2 + 6^2 + 9^2 &= 133 \end{aligned} \right\}$$

39. If $x + y + z = 19$, $x^2 + y^2 + z^2 = 133$, and $xz = y^2$, then the difference between z and x is:

यदि $x + y + z = 19$, $x^2 + y^2 + z^2 = 133$, और $xz = y^2$, तो z और x के बीच का अंतर है:

[A] 5

[B] 3

[C] 6

[D] 4

$$xy + yz + zx = \frac{19^2 - 133}{2}$$

$$xy + yz + y^2 = 14$$

$$y \cdot (x + z + y) = 14$$

$$y = 6$$

$$x + z = 13$$
$$4 + 9$$

$$xz = 36$$
$$4 \times 9$$

$$\begin{array}{r} 36 \\ -133 \\ \hline \end{array}$$



40. If $(a + b - 6)^2 + a^2 + b^2 + 1 + 2b = 2ab + 2a$, then $a = ?$

यदि $(a + b - 6)^2 + a^2 + b^2 + 1 + 2b = 2ab + 2a$, तो $a = ?$

[A] 7

[B] 3.5

[C] 6

[D] 2.5

$$(a+b-6)^2 + [a^2 + b^2 + 1 + 2b - 2ab - 2a] = 0$$

$$(a+b-6)^2 + (-a+b+1)^2 = 0$$

$\downarrow \quad \quad \downarrow$
0 0

$$\begin{array}{rcl} a+b & = & 6 \\ -a+b & = & -1 \\ \hline 2a & = & 7 \\ a & = & \frac{7}{2} \end{array}$$

41. If $(a^2 + 1)(b^2 + 1) + 36 = 12(a + b)$, then $a^3 + b^3 = ?$

यदि $(a^2 + 1)(b^2 + 1) + 36 = 12(a + b)$, तो $a^3 + b^3 = ?$

[A] 198

$$ab=1$$
$$a+b=6$$

[B] 202

[C] 140

[D] 322

$$= 6 \times (6^2 - 3 \times 1)$$
$$= 198$$

$$a^2b^2 + a^2 + b^2 + 1 + 36 - 12a - 12b = 0$$

$$(a^2b^2 + 1 - 2ab) + (a^2 + b^2 + 6^2 - 12a - 12b + 2ab) = 0$$

$$(ab)^2 + 1 - 2ab$$

$$(ab-1)^2 + (a+b-6)^2 = 0$$

$\downarrow$ $\downarrow$
0 0



$$a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2} [(a - b)^2 + (b - c)^2 + (c - a)^2]$$

Example:-

If, $a = 891$

$b = 894$

$c = 899$

तो $a^2 + b^2 + c^2 - ab - bc - ca = ?$

$$= \frac{1}{2} [9 + 25 + 64]$$

$$= \frac{1}{2} \times 98$$

$$= 49$$

$$a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2} [(a - b)^2 + (b - c)^2 + (c - a)^2]$$

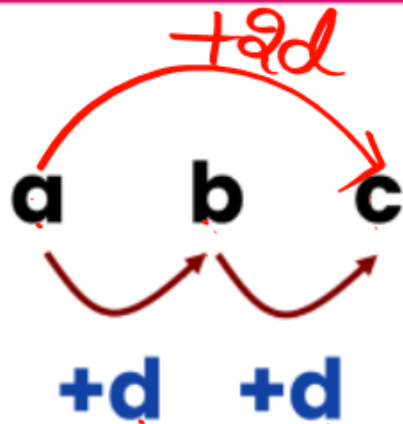
If **a, b & c** are in A.P with common difference **'d'**

तो $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2} [d^2 + d^2 + 4d^2]$

$$= \frac{1}{2} \times \cancel{6}d^2$$

$$= 3d^2$$

$$= 3 \times (\text{common diff})^2$$





$$a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2} [(a - b)^2 + (b - c)^2 + (c - a)^2]$$

Example:-

If, $a =$

71

$b =$

75

$c =$

79

$d = 4$

तो $a^2 + b^2 + c^2 - ab - bc - ca = 3 \times 4^2 = 48$

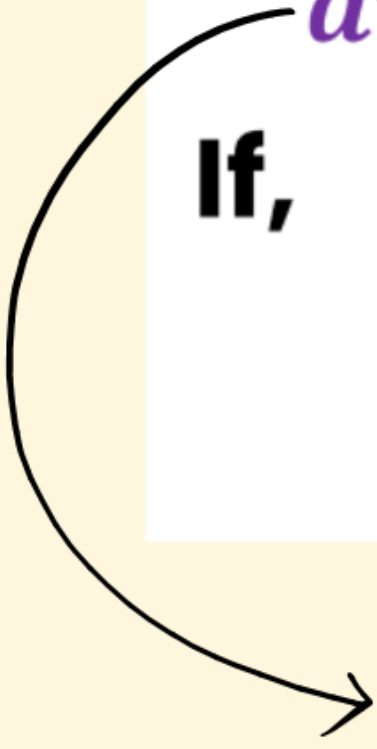

$$a^2 + b^2 + c^2 - ab - bc - ca = 0$$

If,

or

$$a^2 + b^2 + c^2 = ab + bc + ca$$


$$a = b = c$$


$$[(a - b)^2 + (b - c)^2 + (c - a)^2] = 0$$

0

$$a = b$$

0

$$b = c$$

0

$$c = a$$

42. If a, b, c are non-zero real numbers, $abc = 27$ and $a^2 + b^2 + c^2 = ab + bc + ca$ then the value of $\frac{a^2 + 2b^2}{5c} = ?$

यदि a, b, c गैर-शून्य वास्तविक संख्या हैं, $abc = 27$ और $a^2 + b^2 + c^2 = ab + bc + ca$ तब $\frac{a^2 + 2b^2}{5c}$ का मान है ?

~~[A] 1.6~~

~~[C] 1.8~~

[B] 1.5

[D] 1.4

$$a = b = c = 3$$

$$\frac{3a^2}{5c} = \frac{9}{5}$$

43. If $a^2 + b^2 + c^2 = ab + bc + ca$, then the value of $\frac{11a^4 + 13b^4 + 15c^4}{16a^2b^2 + 19b^2c^2 + 17c^2a^2}$ is:

यदि $a^2 + b^2 + c^2 = ab + bc + ca$, तो $\frac{11a^4 + 13b^4 + 15c^4}{16a^2b^2 + 19b^2c^2 + 17c^2a^2}$ का मान क्या है?

[A] $1\frac{3}{4}$

[B] $\frac{1}{4}$

[C] $1\frac{1}{3}$

[D] $\frac{3}{4} = \frac{39}{52}$

$$a = b = c$$

44. If $x \neq 0, y \neq 0$ and $z \neq 0$ and $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx}$, then the relation among x, y, z is:

यदि $x \neq 0, y \neq 0$ और $z \neq 0$ हो, तथा $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx}$ हो, तो x, y और z के बीच संबंध है:

[A] $x + y + z = 0$

[B] $x + y = z$

[C] $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

[D] $x = y = z$

$$\frac{1}{x} = \frac{1}{y} = \frac{1}{z}$$

45. If $(a + b + c)^2 = 3(a^2 + b^2 + c^2)$, then which of the following is necessary true?

[A] $a \neq b = c$

[B] $a = b \neq c$

[C] $a = b = c$

[D] $a \neq b \neq c$

$$\cancel{2(ab + bc + ca)} = \cancel{2(a^2 + b^2 + c^2)}$$

46. If $x + y + z = 4\sqrt{6}$ and $x^2 + y^2 + z^2 = 32$, then $x:y:z = ?$

यदि $x + y + z = 4\sqrt{6}$ और $x^2 + y^2 + z^2 = 32$, तो $x:y:z = ?$

[A] 1:1:1

[B] 1:2:3

[C] 1:3:4

[D] 3:2:1

Special case $\Rightarrow$

$$xy + yz + zx = \frac{(4\sqrt{6})^2 - 32}{2} = 32$$

$$x^2 + y^2 + z^2 = xy + yz + zx$$

$$\Rightarrow x = y = z$$

47. If $x^2 + y^2 + z^2 = 108$ and $x + y + z = 18$, then find $\frac{x-y+z}{x+y-z} = ?$

यदि $x^2 + y^2 + z^2 = 108$ और $x + y + z = 18$ है, तो $\frac{x-y+z}{x+y-z}$ का मान ज्ञात कीजिए।

[A] 6

[B] 3

[C] 1

[D] 2

$$xy + yz + zx = \frac{18^2 - 108}{2} = 108$$

$$x = y = z$$

48. If $x + y + z = 9\sqrt{5}$ and $x^2 + y^2 + z^2 = 135$, then $\frac{x^2 + y^4}{z^3} = ?$

यदि $x + y + z = 9\sqrt{5}$ और $x^2 + y^2 + z^2 = 135$ है, तो $\frac{x^2 + y^4}{z^3} = ?$

✓ [A] $\frac{46\sqrt{5}}{15}$

[B] $\frac{46\sqrt{5}}{5}$

$$= \frac{1 + ae^2}{ae}$$

[C] $\frac{46\sqrt{5}}{9}$

[D] $\frac{46\sqrt{3}}{9}$

$$= \frac{46 \times \sqrt{5}}{3\sqrt{5} \times \sqrt{5}}$$

$$xy + yz + zx = \frac{(9\sqrt{5})^2 - 135}{2} = 135$$

$$x = y = z = 3\sqrt{5}$$

49. If $a = 97.5$, $b = 100$, $c = 102.5$, then find $a^2 + b^2 + c^2 - ab - bc - ca = ?$

यदि $a = 97.5$, $b = 100$, $c = 102.5$ है, तो $a^2 + b^2 + c^2 - ab - bc - ca$ का मान ज्ञात कीजिए।

[A] $\frac{81}{4}$

[C] $\frac{75}{4}$

[B] $\frac{70}{4}$

[D] 15

$$= 3 \times \left(\frac{5}{2}\right)^2 = \frac{75}{4}$$

50. If $p = 106$, $q = 105$ and $r = 104$, then the value of $p^2 + q^2 + r^2 - pq - qr - rp$ is?

यदि $p = 106$, $q = 105$ और $r = 104$, तो $p^2 + q^2 + r^2 - pq - qr - rp$ का मान क्या है?

(SSC SELECTION POST XII GRADUATE LEVEL)

[A] 2

[B] 0

[C] 1

[D] 3

$$= 3 \times 1^2$$

$$d=1$$

51. Find $6637^2 + 6640^2 + 6642^2 - 6637 \times 6640 - 6640 \times 6642 - 6642 \times 6637$?

$6637^2 + 6640^2 + 6642^2 - 6637 \times 6640 - 6640 \times 6642 - 6642 \times 6637$ का मान ज्ञात कीजिए।

[A] 13

[B] 17

[C] 21

[D] 19

$$a = 6637$$

$$b = 6640$$

$$c = 6642$$

$$= \frac{1}{2} [9 + 4 + 25]$$

$$= 19$$

52. If $a - b = 2$ & $b - c = 3$, then find the value of $a^2 + b^2 + c^2 - ab - bc - ca$?

यदि $a - b = 2$ & $b - c = 3$ है, तो $a^2 + b^2 + c^2 - ab - bc - ca$ का मान ज्ञात किजिए?

[A] 20

[B] 18

[C] 19

[D] 31

$a - c = 5$

$$= \frac{1}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$= \frac{1}{2} [4 + 9 + 25]$$